A note on dynamical stabilization of internal spaces in multidimensional cosmology

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23.06.2000

Abstract

The possibility of dynamical stabilization of an internal space is investigated for a multidimensional cosmological model with minimal coupled scalar field as inflaton. It is shown that a successful dynamical compactification crucially depends on the type of interaction between the geometrical modulus field and the inflaton and its decay products. In the considered model a stable compactification can be ensured via trapping of the modulus field by a minimum of the effective potential.

PACS number(s): 04.50.+h, 98.80.Hw

1 Introduction

It is well known that the fundamental physical constants in superstring theories are related to the vacuum expectation values of the dilaton and moduli fields, and variations of these fields would result in variations of the fundamental constants. In the context of standard Kaluza-Klein models moduli are defined by the shape and size of the internal spaces (we shall refer to the corresponding fields as geometrical moduli). Up to now, there are no experiments which show a variation of the fundamental constants (theoretical and experimental bounds on such variations can be found e.g. in [1]). According to observations the internal spaces should be static or nearly static at least from the time of recombination (in some papers arguments are given in favor of the assumption that a variation of the fundamental constants is absent from the time of primordial nucleosynthesis). Therefore, part of any realistic multidimensional model should be a mechanism for moduli stabilization.

Within superstring theories such a stabilization is achieved, e.g., via trapping of the moduli fields by Kähler [2] or racetrack [3] potentials. Recently it was pointed out in Ref. [4], that in a cosmological setting the trapping stabilization of the dilaton field can enhance and become robust due to the coupling of the dilaton to the kinetic energy of ordinary matter fields.

Within multidimensional cosmological models of Kaluza-Klein type the problem of geometrical moduli stabilization by effective potentials was subject of numerous investigations [5]. It was shown that the scale factors of the internal spaces can stabilize, e.g., in pure geometrical models with a bare cosmological constant and curved internal spaces as well as in models with ordinary matter. Small conformal excitations of the internal space metric near the minima of the effective potential have the form of massive scalar fields (gravitational excitons) [6] developing in the external spacetime (later, since the sub-millimeter weak-scale compactification approach these geometrical moduli excitations are also known as radions). Gravexitons

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were investigated for a number of models in Refs. [7]. In Refs. [5] - [7] the 4-dimensional Planck scale $M_{Pl(4)}$ was implicitly understood as the $D-$dimensional fundamental scale[1] and it was assumed that the internal spaces are compactified at sizes somewhere between the Planck scale $L_{Pl} \sim 10^{-33}$cm and the Fermi scale $L_F \sim 10^{-17}$cm to make them unobservable.

Recently it has been realized that the higher-dimensional fundamental scale $M_{Pl(D)}$ can be lowered from the 4-dimensional Planck scale $M_{Pl(4)} = 1.22 \times 10^{19}$ GeV down to the Standard Model electroweak scale $M_{Pl(4+D')} \sim M_{EW} \sim 1$ TeV providing by this way a new scenario for the resolution of the hierarchy problem. The corresponding proposal led to an intensive study of various multidimensional theories. The considered models can be roughly devided into two topological classes.

The first class consists of models with warped products of Einstein spaces as internal spaces. For simplicity, the corresponding scale (warp) factors are usually assumed as depending only on the coordinates of the external spacetime. Whereas gravitational interactions in such models can freely propagate in all multidimensional (bulk) space, the Standard Model (SM) matter is localized on a 3-brane with thickness of order of the Fermi length in the extra dimensions. Such a model was used in Ref. [8] for the demonstration of the basic features of the sub-millimeter weak-scale compactification hypothesis with $M_{Pl(D)} \sim M_{EW}$. Different aspects of geometrical moduli stabilization for such models were considered e.g. in Refs. [9]. A comparison of effective cosmological constant and gravexciton masses arising in the electroweak fundamental scale approach with those in a corresponding Planck scale approach was given in Ref. [10].

The second class consists of models following from Hořava-Witten theory [11] where one starts from the strongly coupled regime of $E_8 \times E_8$ heterotic string theory and interprets it as M-theory on an orbifold $\mathbb{R}^{10} \times S^1 / \mathbb{Z}_2$ with a set of E_8 gauge fields at each ten-dimensional orbifold fixed plane. After compactification on a Calabi-Yau three-fold and dimensional reduction one arrives at effective $5-$dimensional solutions which describe a pair of parallel 3-branes with opposite tension, and location at the orbifold planes. For these models the $5-$dimensional metric contains a $4-$dimensional metric component multiplied by a warp factor which is a function of the additional dimension [12]. The geometrical moduli stabilization for such models was considered, e.g., in Refs. [13]. Difficulties of the stabilization in such models connected with the required unrealistic fine-tuning of the equation of state on our 3-brane were pointed out in Ref. [14], with possible resolutions proposed in [15]. Moreover, in Refs. [16] it was shown that the stabilization of the extra dimension is a necessary condition for the correct transition from $5-$dimensional models with branes to the standard $4-$dimensional Friedmann cosmology. Various other aspects of cosmological brane world scenarios were investigated, e.g., in Refs. [17].

Usually, the moduli stabilization is based on the trapping of the geometrical moduli fields at a minimum of an effective potential so that the fields are static (or may at most oscillate near this minimum due to quantum fluctuations). However, in absence of any minima, nothing forbids them to evolve very slowly as long as their evolution does not contradict the observable data. It is clear that a sufficiently slow evolution is allowed, if the moduli fields came into this regime before primordial nucleosynthesis. Such an approach is in spirit of Paul Steinhardt's quintessence scenario [18]. It may happen that moduli fields asymptotically tend to some limit. We shall call such a behavior dynamical stabilization. An example for a dynamical stabilization in modular cosmology was presented in Ref. [19]. However, the interaction with ordinary matter fields can destroy this stabilization mechanism. The situation occurs, e.g., in the model considered in sections 2, and 3 of the present paper.

Subject of the investigation in the present paper is a multidimensional cosmological model with a minimal coupled scalar field as inflaton field. In Ref. [21] it was pointed out that a dynamical stabilization of the geometrical modulus could be possible for a model with one Ricci-flat internal space and a zero bulk cosmological constant. Here, we investigate this model in more detail and show that the interaction of the geometrical modulus field with the inflaton field in most cases destroys the dynamical stabilization and leads to decompactification of the extra dimension. There are only two possible ways for a stable compactification of the extra dimension. For the first one, a dynamical stabilization, one has to assume that the modulus field is only coupled to the inflaton field but not to its decay products. Due to the exponential decay of the inflaton during reheating the force term in the modulus equation, obtained from the effective potential of the model, decreases also exponentially. The present friction term provides then the stabilization of the modulus field. The second possibility for a stabilization consists in the trapping of the modulus field near a minimum of the effective potential. As simple example we consider the effective potential from Ref. [10] where the minimum is generated by a non-Ricci-flat (curved) internal space and a non-zero bare cosmological constant. The analysis does not depend on the choice of the $D-$dimensional fundamental scale.

The paper is organized as follows. In section 2 we explain the general setup of our model and show a possible mechanism for a dynamical stabilization of the geometrical modulus in zero-order approximation. In section 3 we investigate the dynamical behavior of the modulus and the inflaton field in more detail and show that the interaction between them in most cases results in a decompactification of the internal space. An example for a stable compactification of the internal space by trapping of the modulus field near the minimum of the effective potential is presented in section 4. The brief Conclusions of the paper (section 5) are followed by an Appendix on higher dimensional perfect fluid potentials and specific features of their dimensional reduction.

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[1]Hereafter, $D = D'+4$ is the total dimension of the multidimensional spacetime, D' — the dimension of the internal components.

2 Model and general setup

We consider a cosmological model with metric

$$g = g^{(0)} + e^{2\beta(x)}g^{(1)} \equiv g^{(0)} + b^2(x)g^{(1)}\,, \tag{2.1}$$

which is defined on a manifold with warped product topology[2]

$$M = M_0 \times M_1\,, \tag{2.2}$$

where x are some coordinates of the $D_0 = (d_0 + 1)$ - dimensional manifold M_0 and

$$g^{(0)} = g^{(0)}_{\mu\nu}(x)dx^\mu \otimes dx^\nu. \tag{2.3}$$

(The model corresponds to the type one class discussed in the Introduction.)

With $D = D_0 + d_1$ as total dimension, κ_D^2 a $D-$dimensional gravitational constant and S_{YGH} the standard York - Gibbons - Hawking boundary term we choose the action functional in the form

$$S = \frac{1}{2\kappa_D^2}\int_M d^D x \sqrt{|g|}R[g] - \frac{1}{2}\int_M d^D x \sqrt{|g|}\left(g^{MN}\partial_M\tilde\chi\partial_N\tilde\chi + 2U(\tilde\chi)\right) + S_{YGH}\,, \tag{2.4}$$

where the minimal coupled scalar field $\tilde\chi$ with an arbitrary potential $U(\tilde\chi)$ depends on the external coordinates x only. This field can be understood as a zero mode of a bulk field. From the other hand, such a scalar field can naturally originate also in non-linear $D-$dimensional theories [22] where the metric ansatz (2.1) ensures its dependence on the x coordinate only.

The main goal of the present paper consists in an investigation of the internal space dynamical stabilization. As it was shown in [21], such a possibility exists for the considered model only if a bare $D-$dimensional cosmological constant Λ identically equals zero and the internal space is a Ricci-flat one: $R\left[g^{(1)}\right] = 0$.

Let $b_0 = L_{Pl}e^{\beta_0}$ be the compactification scale of the internal space at the present time and

$$v_{d_1} \equiv v_0 \times v_i \equiv b_0^{d_1} \times \int_{M_1} d^{d_1}y\sqrt{|g^{(1)}|} \tag{2.5}$$

the corresponding total volume of the internal space. Instead of β it is convenient to introduce the shifted quantity:

$$\tilde\beta = \beta - \beta_0\,. \tag{2.6}$$

Then, after dimensional reduction action (2.4) reads

$$\begin{aligned}
S &= \frac{1}{2\kappa_0^2}\int_{M_0} d^{D_0}x\sqrt{|g^{(0)}|}e^{d_1\tilde\beta}\left\{R\left[g^{(0)}\right] - d_1(1-d_1)g^{(0)\mu\nu}\partial_\mu\tilde\beta\,\partial_\nu\tilde\beta - g^{(0)\mu\nu}\kappa_D^2\partial_\mu\tilde\chi\partial_\nu\tilde\chi - 2\kappa_D^2 U(\tilde\chi)\right\} \\
&= \int_{M_0} d^{D_0}x\sqrt{|g^{(0)}|}\left\{\Phi R\left[g^{(0)}\right] - \frac{1-d_1}{d_1}g^{(0)\mu\nu}\frac{\partial_\mu\Phi\,\partial_\nu\Phi}{\Phi} - \kappa_0^2\Phi g^{(0)\mu\nu}\partial_\mu\chi\partial_\nu\chi - 2\kappa_0^2\Phi V(\chi)\right\}\,, \tag{2.7}
\end{aligned}$$

where

$$\Phi = \frac{1}{2\kappa_0^2}e^{d_1\tilde\beta} = \frac{1}{2\kappa_0^2}\left(\frac{b}{b_0}\right)^{d_1} \tag{2.8}$$

and we redefined the scalar field $\tilde\chi$ and its potential as follows:

$$\chi = \sqrt{v_{d_1}}\,\tilde\chi\,, \quad V(\chi) = v_{d_1}U\left(\chi/\sqrt{v_{d_1}}\right)\,. \tag{2.9}$$

In action (2.7) κ_0^2 is the D_0-dimensional (4-dimensional) gravitational constant:

$$\kappa_0^2 = \frac{8\pi}{M_{Pl}^2} := \frac{\kappa_D^2}{v_{d_1}}\,, \tag{2.10}$$

where $M_{Pl} = M_{Pl(4)} = 1.22 \times 10^{19}$GeV. It is clear that the scale of the internal space compactification b_0 is defined now by the energetic scale of the $D-$dimensional gravitational constant κ_D^2. If we normalize κ_D^2 in such a way that $\kappa_D^2 = 8\pi/M_{EW}^{2+d_1}$, where $M_{EW} \sim 1$TeV is the SM electroweak scale, then we arrive at

the intensively discussed estimate [8]: $b_0 \sim 10^{(32/d_1)-17}$cm. (Here, we assume for a toroidal internal space $v_i \equiv 1$.) In the present paper we shall not discuss different choices of κ_D^2. It has no influence on the main conclusions about the possibility of an internal space dynamical stabilization. Generally speaking, we might not fix a scale of the internal space stabilization in Eqs. (2.6) - (2.8), but simply set there $\beta_0 = 0$ and $b_0 = L_{Pl}$. For our concret model this does not mean that the stabilization takes place at the planckian scale but results simply in a planckian normalization of the $D-$dimensional gravitational constant: $\kappa_D^2 \sim M_{Pl}^{-(2+d_1)}$ [10]. In the latter approach, the scale of stabilization is defined from the equations of motion.

To get the equations of motion in the Brans-Dicke frame it is convenient to rewrite action (2.7) as follows:

$$S = \int\limits_{M_0} d^{D_0}x \sqrt{|g^{(0)}|} \left\{ \Phi R\left[g^{(0)}\right] - \frac{\omega}{\Phi} g^{(0)\mu\nu} \partial_\mu \Phi \, \partial_\nu \Phi + 2\kappa_0^2 \Phi L_m \right\}, \tag{2.11}$$

where $\omega := (1 - d_1)/d_1 < 0$ and

$$L_m = -\frac{1}{2} g^{(0)\mu\nu} \partial_\mu\chi \partial_\nu\chi - V(\chi). \tag{2.12}$$

Varying this action with respect to the metric $g_{\mu\nu}^{(0)}$ and the fields Φ and χ, we get respectively

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}^{(0)} R = \kappa_0^2 T_{\mu\nu} + \frac{\omega}{\Phi^2}\left[\Phi_{;\mu}\Phi_{;\nu} - \frac{1}{2} g_{\mu\nu}^{(0)} \Phi_{;\lambda}\Phi^{;\lambda}\right] + \frac{1}{\Phi}\left[\Phi_{;\mu;\nu} - g_{\mu\nu}^{(0)}\Box\Phi\right], \tag{2.13}$$

$$R - \frac{\omega}{\Phi^2} g^{(0)\,\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + 2\kappa_0^2 L_m + \frac{2\omega}{\Phi}\Box\Phi = 0 \tag{2.14}$$

and

$$\Box\chi + \frac{1}{\Phi} g^{(0)\,\mu\nu}\partial_\mu\Phi\partial_\nu\chi = V', \tag{2.15}$$

where

$$\begin{aligned}
T_{\mu\nu} &= g_{\mu\nu}^{(0)} L_m + \partial_\mu\chi\partial_\nu\chi, \\
T &\equiv T_\mu^\mu = D_0 L_m + g^{(0)\,\mu\nu}\partial_\mu\chi\partial_\nu\chi,
\end{aligned} \tag{2.16}$$

$$\Box \equiv \frac{1}{\sqrt{|g^{(0)}|}} \partial_\mu \left(\sqrt{|g^{(0)}|} g^{(0)\,\mu\nu} \partial_\nu \right) \tag{2.17}$$

and the prime denotes the derivative with respect to the field χ.

Contracting (2.13) with respect to $g^{(0)\,\mu\nu}$ yields

$$R = \frac{\kappa_0^2}{(2 - D_0)/2} T + \frac{\omega}{\Phi^2} g^{(0)\,\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + \frac{1}{\Phi}\frac{D_0 - 1}{(D_0 - 2)/2}\Box\Phi. \tag{2.18}$$

Now, combining (2.14) and (2.18) we can rewrite the equation of motion for the field Φ in the convenient form

$$\begin{aligned}
\frac{1}{\Phi}\left(2\omega + \frac{D_0 - 1}{(D_0 - 2)/2}\right)\Box\Phi &= \frac{2\kappa_0^2}{D_0 - 2}T - 2\kappa_0^2 L_m \\
&= -\frac{4\kappa_0^2}{D_0 - 2}V(\chi).
\end{aligned} \tag{2.19}$$

Let us specify metric $g^{(0)}$ as

$$g^{(0)} = -dt \otimes dt + a^2(t)\bar{g}^{(0)}, \tag{2.20}$$

where $\bar{g}^{(0)}$ is the metric of the d_0-dimensional Ricci-flat external space: $R\left[\bar{g}^{(0)}\right] = kd_0(d_0-1) = 0 \implies k = 0$ (in accordance with recent observations our Universe is flat). For this metric we get

$$R_{00}[g^{(0)}] = -d_0\frac{\ddot{a}}{a} \quad \text{and} \quad R[g^{(0)}] = d_0\left[2\frac{\ddot{a}}{a} + (d_0 - 1)\frac{\dot{a}^2}{a^2}\right], \tag{2.21}$$

where the dot denotes the derivative with respect to the synchronous time t in the Brans-Dicke (BD) frame (string frame).

Below we consider homogeneous fields: $\Phi = \Phi(t)$ and $\chi = \chi(t)$. Then, the $00-$component of the Einstein eq. (2.13) reads

$$H^2 + \frac{2}{d_0 - 1} H\frac{\dot{\Phi}}{\Phi} = \frac{\omega}{d_0(d_0 - 1)}\left(\frac{\dot{\Phi}}{\Phi}\right)^2 + \frac{2\kappa_0^2}{d_0(d_0 - 1)}\left(\frac{1}{2}\dot{\chi}^2 + V\right), \tag{2.22}$$

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where $H \equiv \dot{a}/a$ is the Hubble parameter. If we take into account that for our metric (2.20) and any homogeneous field ϕ holds $\Box\phi = -\ddot{\phi} - d_0 H\dot{\phi}$, then we get for equations (2.19) and (2.15) correspondingly:

$$\ddot{\Phi} + d_0 H\dot{\Phi} = \frac{2\kappa_0^2}{\omega(d_0 - 1) + d_0}\Phi V(\chi) \tag{2.23}$$

and

$$\ddot{\chi} + d_0 H\dot{\chi} + \frac{\dot{\Phi}}{\Phi}\dot{\chi} = -V'. \tag{2.24}$$

We suppose that the potential $V(\chi)$ of our model has a zero minimum: $V|_{min} = 0$. Then, after the χ field has evoluted down-hill to this minimum and is frozen out, the right hand side of eq. (2.23) is equal to zero and the equation has a solution

$$\Phi(t) = \frac{1}{2\kappa_0^2} - \dot{\Phi}_0 \int\limits_t^\infty dt\, e^{-d_0 \int\limits_{t_0}^t H\,dt}, \tag{2.25}$$

where $\dot{\Phi}_0 := \dot{\Phi}(t = t_0)$ is an initial value for $\dot{\Phi}$. The constant of integration in (2.25) is chosen in such a way that $\Phi(t \to \infty) \longrightarrow 1/2\kappa_0^2$, i.e. that it corresponds to the stabilization of internal space $b \longrightarrow b_0$. Thus, in zero-order approximation (with respect to the interaction between the Φ and χ fields in eq. (2.23)) the dynamical stabilization of the internal space takes place if the integral in eq. (2.25) is convergent at the upper limit $t \to \infty$. If the integral diverges the dynamical stabilization mechanism fails to work and decompactification occurs. A detailed analysis of this problem will be given in the next section where we use a self-consistent approach and investigate the influence of the interaction between the fields on the dynamical stabilization of the modulus field Φ. It is more convenient to perform the corresponding analysis in the Einstein frame. Obviously, if stabilization occurs in the Einstein frame it has place also in the Brans-Dicke frame and vice versa.

3 The Einstein frame

A conformal transformation to the Einstein frame is given by equation

$$g_{\mu\nu}^{(0)} = \Omega^2 \tilde{g}_{\mu\nu}^{(0)} := \left(e^{d_1\tilde{\beta}} \right)^{-\frac{2}{D_0 - 2}} \tilde{g}_{\mu\nu}^{(0)} = \left(2\kappa_0^2 \Phi \right)^{-\frac{2}{D_0 - 2}} \tilde{g}_{\mu\nu}^{(0)} \tag{3.1}$$

and yields [6]

$$S = \frac{1}{2} \int\limits_{M_0} d^{D_0}x \sqrt{|\tilde{g}^{(0)}|} \left\{ \frac{1}{\kappa_0^2} \tilde{R}\left[\tilde{g}^{(0)} \right] - \tilde{g}^{(0)\mu\nu}\partial_\mu\varphi\,\partial_\nu\varphi - \tilde{g}^{(0)\mu\nu}\partial_\mu\chi\,\partial_\nu\chi - 2U_{eff}(\varphi,\chi) \right\}, \tag{3.2}$$

where

$$\varphi := \pm\frac{1}{\kappa_0}\sqrt{\frac{d_1(D-2)}{D_0 - 2}}\tilde{\beta} \tag{3.3}$$

and the effective potential can be written as

$$U_{eff}[\varphi,\chi] := e^{2\kappa_0\varphi\sqrt{\frac{d_1}{(D-2)(D_0-2)}}}V(\chi) := e^{2\sigma\kappa_0\varphi}V(\chi). \tag{3.4}$$

(For definiteness we use the minus sign in eq. (3.3).)

The dimensionally reduced action (3.2) describes a system of 4-dimensional gravitational and scalar fields. The problem of the internal space stabilization is reduced now to the investigation of the dynamics of these fields. For this purpose we specify the metric $\tilde{g}^{(0)}$. In the Einstein frame the 4-dimensional metric (2.20) can be rewritten as

$$\tilde{g}^{(0)} = -d\tilde{t} \otimes d\tilde{t} + \tilde{a}^2(\tilde{t})\bar{g}^{(0)}, \tag{3.5}$$

where 'tilded' quantities are related to the Einstein frame.

Then, the equations of motion following from action (3.2) read:

$$\tilde{H}^2 \equiv \left(\frac{\dot{\tilde{a}}}{\tilde{a}}\right)^2 = \frac{2\kappa_0^2}{d_0(d_0 - 1)}\left\{ \frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}\dot{\chi}^2 + U_{eff} \right\}, \tag{3.6}$$

$$\ddot{\varphi} + d_0\tilde{H}\dot{\varphi} = -\frac{\partial U_{eff}}{\partial\varphi} = -2\kappa_0\sigma e^{2\sigma\kappa_0\varphi}V(\chi), \tag{3.7}$$

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$$\ddot{\chi} + d_0 \tilde{H}\dot{\chi} = -\frac{\partial U_{eff}}{\partial \chi} = -e^{2\sigma\kappa_0\varphi}V'(\chi)\,. \tag{3.8}$$

Hereafter, dots denote derivatives with respect to the synchronous time $\tilde{t}$ in the Einstein frame, and primes denote derivatives with respect to the inflaton field χ. Further on, we assume for the external space dimension the usual value $d_0 = 3$. For this choice of d_0 the parameter σ is fixed by the dimension of the extra space as $\sigma = \sqrt{d_1/2(d_1+2)}$.

Inflation in our particular model (3.6) - (3.8) is known as soft inflation and was extensively studied in Refs. [20, 21]. Here we simply quote the corresponding results. During inflation the slow roll conditions $\dot{\varphi}^2 \ll 2U_{eff}$, $\dot{\chi}^2 \ll 2U_{eff}$, $\ddot{\varphi} \ll 3\tilde{H}\dot{\varphi}$ and $\ddot{\chi} \ll 3\tilde{H}\dot{\chi}$ hold and the Eqs. (3.6) - (3.8) reduce to the simpler form

$$
\begin{aligned}
3\tilde{H}\dot{\varphi} &\approx -2\sigma\kappa_0 e^{2\sigma\kappa_0\varphi}V(\chi)\,,\\
3\tilde{H}\dot{\chi} &\approx -e^{2\sigma\kappa_0\varphi}V'(\chi)\,,\\
\tilde{H}^2 &\approx \frac{\kappa_0^2}{3}U_{eff} = \frac{\kappa_0^2}{3}e^{2\sigma\kappa_0\varphi}V(\chi)\,.
\end{aligned}
\tag{3.9}
$$

As result, the fields χ and φ are connected with the scale factor of the external space by the relations

$$\dot{\varphi} \approx -\frac{2\sigma}{\kappa_0}\tilde{H}$$

$$\implies \quad \kappa_0(\varphi - \varphi_0) \approx -2\sigma\int \tilde{H}d\tilde{t} = -2\sigma\ln\frac{\tilde{a}}{\tilde{a}_0} \tag{3.10}$$

and

$$\kappa_0^2 \int_{\chi_0}^{\chi} d\chi \frac{V}{V'} = \ln\frac{\tilde{a}}{\tilde{a}_0}\,, \tag{3.11}$$

where the initial conditions are denoted by the subscript "0" and are fixed at $\tilde{t}_0$, when the Universe enters the inflationary phase. From (3.3), (3.10) we see that during the inflationary stage of the external spacetime the scale factor of the internal space undergoes inflation too (see e.g. also [21])

$$b(\tilde{t}) = \bar{b}_0 e^{\frac{2}{D-2}\int \tilde{H}d\tilde{t}}\,. \tag{3.12}$$

The necessity of such a stage was stressed in paper [23], where it was shown that to solve the horizon and flatness problem, there must be a stage of inflation in the bulk space before the compactification of the internal spaces can be completed. If we take into account that during this stage $\tilde{a} \sim \exp\left(\int \tilde{H}d\tilde{t}\right)$ then we get following relation between initial and final values: $b_f/b_i = [a_f/a_i]^{2/(D-2)}$ (see also [21]).

Let us investigate now the dynamical behavior of the model at the stage when the inflaton scalar field χ is evolving down-hill to the minimum of its effective potential and is located in the very vicinity of this minimum. Without loss of generality, we suppose that potential $V(\chi)$ has a zero minimum at the point $\chi = 0$: $V|_{\chi=0} = 0$, $V'|_{\chi=0} = 0$, $V''|_{\chi=0} > 0$. This leads to two implications.

First, for $|\chi| < 1$ (i.e. $|\chi| < M_{Pl}$) this potential can be crudely approximated by

$$V(\chi) \approx \frac{1}{2}m_\chi^2\chi^2\,, \tag{3.13}$$

where $m_\chi^2 \equiv V''|_{\chi=0}$. In the same approximation we get for eqs. (3.7) and (3.8) correspondingly:

$$\ddot{\varphi} + 3\tilde{H}\dot{\varphi} + \kappa_0\sigma e^{2\sigma\kappa_0\varphi}m_\chi^2\chi^2 = 0 \tag{3.14}$$

and

$$\ddot{\chi} + 3\tilde{H}\dot{\chi} + e^{2\sigma\kappa_0\varphi}m_\chi^2\chi = 0\,. \tag{3.15}$$

Second, from the structure of the effective potential $U_{eff}(\chi,\varphi) = e^{2\sigma\kappa_0\varphi}V(\chi)$ it is clear that the zero minimum $V|_{\chi=0} = 0$ is globally degenerate and, in crude analogy with Goldstone bosons in φ^4-theories, the field φ plays the role of the zero mode along the degeneration line $\chi = 0$ in the $(\chi,\varphi)-$plane. This is easy to see from the Hessian of the effective potential

$$\begin{pmatrix} \partial^2_{\varphi\varphi} & \partial^2_{\varphi\chi} \\ \partial^2_{\chi\varphi} & \partial^2_{\chi\chi} \end{pmatrix} U_{eff}|_{\chi=0} = \begin{pmatrix} 0 & 0 \\ 0 & e^{2\sigma\kappa_0\varphi}m_\chi^2 \end{pmatrix}\,, \tag{3.16}$$

which defines the mass matrix of the normal modes of the model at the point $\chi = 0$ (see also Ref. [6]). This means that the geometrical modulus field φ corresponds to a flat direction of the effective potential U_{eff} and is not stabilized by a minimum of this potential (see e.g. Ref. [24]). Due to the remaining $\varphi-$dependence of the effective mass of the $\chi-$mode the analogy with Goldstone bosons in φ^4-theories is only very crude.

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As first step of our analysis, we investigate the equation of motion (3.14) of the modulus field not taking into account the energy constraint (3.6) and assuming that the χ field is frozen in the minimum of the potential $V(\chi)$ (zero-order approximation in the fluctuations of field χ):

$$\chi \longrightarrow 0 \quad \Longrightarrow \quad \ddot{\varphi} + 3\tilde{H}\dot{\varphi} \approx 0 \, . \tag{3.17}$$

For a cosmologically non-damped evolution with $\tilde{H} = 0$ we would have $\ddot{\varphi} \approx 0$ and a dynamically non-stabilized behavior of the modulus field $\varphi = \dot{\varphi}_0 \tilde{t} + \varphi_0$, where the initial values are chosen at some $\tilde{t} = \tilde{t}_0$: $\dot{\varphi}_0 := \dot{\varphi}(\tilde{t} = \tilde{t}_0), \quad \varphi_0 := \varphi(\tilde{t} = \tilde{t}_0)$. Due to the damping term $3\tilde{H}\dot{\varphi} \neq 0$ eq. (3.17) has a solution

$$\varphi(\tilde{t}) = -\dot{\varphi}_0 \int_{\tilde{t}}^{\infty} d\tilde{t} e^{-3 \int_{\tilde{t}_0}^{\tilde{t}} \tilde{H} d\tilde{t}} \, , \tag{3.18}$$

which describes a dynamical (asymptotical) stabilization if the integral converges at its upper limit $\tilde{t} \to \infty$. The constant of integration in (3.18) is chosen in such a way that $\varphi(\tilde{t} \to \infty) \longrightarrow 0$ and corresponds to the internal space stabilization at $b \longrightarrow b_0$. Obviously, this solution is the Einstein frame analogue of the Brans-Dicke frame solution (2.25) and reproduces the dynamical stabilization scenario of Ref. [21]. For a power-law behavior of the external scale factor: $\tilde{a} \sim \tilde{t}^s \Longrightarrow \tilde{H} = \dot{\tilde{a}}/\tilde{a} = s/\tilde{t}$ ($s = 1/2 \, , 2/3$ corresponds to the radiation dominated (RD) and matter dominated (MD) era respectively) this equation yields

$$\varphi(\tilde{t}) = -\frac{\dot{\varphi}_0}{3s - 1} \frac{\tilde{t}_0^{3s}}{\tilde{t}^{3s-1}} \, , \qquad s > \frac{1}{3} \, . \tag{3.19}$$

We can use this equation for estimates of the variation of the fundamental constants due to the dynamics of the internal space. Usually [1], such variations are proportional to $\dot{b}/b$. From eq. (3.19) we get

$$\frac{\dot{b}}{b} = -\sqrt{\frac{D_0 - 2}{d_1(D - 2)}} \kappa_0 \dot{\varphi}_0 \left(\frac{\tilde{t}_0}{\tilde{t}} \right)^{3s} \, . \tag{3.20}$$

We choose the initial values $\dot{\varphi}_0$ and $\tilde{t}_0$ in such way that they correspond to the end of inflation: $\kappa_0 \dot{\varphi}_0 = -2\sigma \tilde{H}_e$ (see eq. (3.10)) and $\tilde{H}_e \tilde{t}_0 \sim 70$. Then, eq. (3.20) yields

$$\frac{\dot{b}}{b} \approx \frac{2}{D - 2} \tilde{H}_e \left(\frac{70}{\tilde{H}_e \tilde{t}} \right)^{3s} \, . \tag{3.21}$$

For the simplest models of inflation, as in our case, the COBE data predict $\tilde{H}_e \sim 10^{-5} M_{Pl}$. Thus, we obtain $\dot{b}/b\big|_{\tilde{t} \sim 10^3 sec \, ; s = 1/2} \sim 10^{-14} \mathrm{yr}^{-1}$. This means that effectively there is no variation of the fundamental constants starting from the time of nucleosynthesis. For the modulus field value we can easily get an estimate:

$$\kappa_0 \varphi(\tilde{t}) \approx \frac{140\sigma}{3s - 1} \left(70/\tilde{H}_e \tilde{t} \right)^{3s-1} \Longrightarrow \kappa_0 \varphi\big|_{\tilde{t} \sim 10^3 sec \, ; s = 1/2} \sim 10^{-18} \ll 1 \, . \tag{3.22}$$

Thus, in zero order approximation and not taking account of the constraint (3.6), the internal space would appearently stabilize to this time.

Let us now use a self-consistent approach to our model taking also into account the constraint (3.6). For frozen χ field we have $\chi = \chi_0 = 0$, $\dot{\chi}_0 = 0$ and $V(\chi_0) = 0$ so that the energy density $\rho_{\chi(E)} \equiv \frac{1}{2}\dot{\chi}^2 + U_{eff}$ vanishes ($\rho_{\chi(E)}(\chi_0) = 0$) and the equation system (3.6) - (3.8) reads:

$$\tilde{H}^2 = \frac{\kappa_0^2}{6}\dot{\varphi}^2 \, , \tag{3.23}$$

$$\ddot{\varphi} + 3\tilde{H}\dot{\varphi} = 0 \, . \tag{3.24}$$

The solutions are easily found as

$$\tilde{H} = \frac{1}{3\tilde{t}}, \quad \tilde{a} \sim \tilde{t}^{1/3}, \quad \kappa_0 \dot{\varphi} = \pm \sqrt{\frac{2}{3}} \frac{1}{\tilde{t}}, \quad \kappa_0 \varphi = \pm \sqrt{\frac{2}{3}} \ln \frac{\tilde{t}}{\tilde{t}_0} + \kappa_0 \varphi_0 \tag{3.25}$$

and show that the possible dynamical stabilization mechanism, which holds for $s > 1/3$, fails to work in the case $s = 1/3$ due to the logarithmically diverging φ. Heuristically, we can say that for frozen inflaton field χ the decoupled modulus field φ behaves like an ultra-stiff perfect fluid[3] with equation of state

---

[3] See [25, 26] and the Appendix for a discussion of the equivalence between a scalar field φ and a perfect fluid with energy density ρ_φ, pressure P_φ and equation of state $P_\varphi = (\alpha - 1)\rho_\varphi$.

$P_{\varphi(E)} = \rho_{\varphi(E)} \sim \tilde{a}^{-6}$ and produces not enough "cosmological friction" $\tilde{H} = s/\tilde{t}$ in order to come to a rest at some finite value $|\varphi(\tilde{t} \to \infty)| < \infty$. As consequence the additional dimensions cannot stabilize, but rather they decompactify.

This behavior can be partially circumvented by passing from the homogeneous modulus approach $\varphi = \varphi(\tilde{t})$ to an inhomogeneous one $\varphi = \varphi(\tilde{x})$ with modulus fluctuations behaving like radiation (see e.g. [19, 27]). The corresponding energy density could lead to the needed "cosmological friction" $\tilde{H}$ with $s > 1/3$. But as we will show below in this section, the interaction of the modulus field with the inflaton will destroy such a dynamical stabilization mechanism. For simplicity, we will restrict our subsequent considerations to purely homogeneous fields.

For such fields we can circumvent the decompactification mechanism (3.25) if we assume that χ performs small fluctuations around the minimum of its potential $V(\chi)$ yielding by this way a nonvanishing energy density $\rho_{\chi(E)} > 0$ which could provide the needed "cosmological friction" $\tilde{H}$ with $s > 1/3$. So, we shall investigate corrections to the equation system (3.6), (3.14), (3.15) due to the dynamics of the field χ. To achieve this goal, we assume that the modulus field φ is already nearly stabilized at $\varphi \approx 0$ and embed our system (3.14), (3.15) in a generalized astrophysical setting allowing for decay processes of the inflaton field χ into usual matter. The corresponding particle interactions result in a polarization operator Π of the field χ which shifts the squared mass in eq. (3.15): $m_\chi^2 \longrightarrow m_\chi^2 + \Pi$ (see e.g. [28]). The imaginary part of Π is responsible for the inflaton decay and is connected with the decay rate Γ_χ by the relation $\mathrm{Im}\Pi = m_\chi \Gamma_\chi$. It is shown in Ref. [28] that phenomenologically such a decay can be taken into account by adding an extra friction term $\Gamma\dot{\chi}$ to the classical equation of motion (3.15) (instead of adding the term proportional to the imaginary part of the polarization operator):

$$\ddot{\chi} + (3\tilde{H} + \Gamma_\chi)\dot{\chi} + e^{2\sigma\kappa_0\varphi}m_\chi^2\chi = 0\,. \tag{3.26}$$

We use this equation to crudely understand the dynamical behavior of the field χ during the post-inflationary evolution of the Universe (not taking into account preheating or subtleties of the decay processes [28]). It is convenient to investigate eq. (3.26) with the help of a substitution (see [29], Chapter 14):

$$\chi(\tilde{t}) := B(\tilde{t})u(\tilde{t}) := e^{-\frac{1}{2}(\Gamma_\chi\tilde{t}+3\int\tilde{H}d\tilde{t})}u(\tilde{t})\,, \tag{3.27}$$

where the function $u(\tilde{t})$ satisfies the equation

$$\ddot{u} + \left[\bar{m}_\chi^2 - \frac{1}{4}(3\tilde{H} + \Gamma_\chi)^2 - \frac{3}{2}\dot{\tilde{H}}\right]u = 0 \tag{3.28}$$

and $\bar{m}_\chi^2(\tilde{t}) = m_\chi^2 \exp(2\sigma\kappa_0\varphi)$. If we suppose further that $(3\tilde{H} + \Gamma_\chi)^2, \dot{\tilde{H}}, (\dot{\bar{m}}_\chi/\bar{m}_\chi)^2$ are small compared with $\bar{m}_\chi^2$ (which, for a viable model should take place in the large $\tilde{t} \gg t_0$ limit when $|\kappa_0\varphi| \ll 1$), then eq. (3.28) has an approximate solution of the form (see also [30])

$$u(\tilde{t}) \approx \cos(\bar{m}_\chi\tilde{t}) \quad \Longrightarrow \quad \chi(\tilde{t}) \approx B(\tilde{t})\cos(\bar{m}_\chi\tilde{t})\,. \tag{3.29}$$

It can be easily seen from the definition of $B(\tilde{t})$ that this function satisfies the equation

$$\frac{d}{d\tilde{t}}\left(\tilde{a}^3 B^2\right) = -\Gamma_\chi\tilde{a}^3 B^2\,. \tag{3.30}$$

Approximating the energy density of the inflaton and the corresponding number density as

$$\rho_{\chi(E)} = \frac{1}{2}\dot{\chi}^2 + \frac{1}{2}\bar{m}_\chi^2\chi^2 \approx \frac{1}{2}B^2\bar{m}_\chi^2\,, \quad n_{\chi(E)} \approx \frac{1}{2}B^2\bar{m}_\chi \tag{3.31}$$

shows that they satisfy the differential relations

$$\frac{d}{d\tilde{t}}(\tilde{a}^3\rho_{\chi(E)}) = -\Gamma_\chi\tilde{a}^3\rho_{\chi(E)} \qquad \text{and} \qquad \frac{d}{d\tilde{t}}(\tilde{a}^3 n_{\chi(E)}) = -\Gamma_\chi\tilde{a}^3 n_{\chi(E)} \tag{3.32}$$

with solutions $\rho_{\chi(E)} \sim e^{-\Gamma_\chi\tilde{t}}\tilde{a}^{-3}$ and $n_{\chi(E)} \sim e^{-\Gamma_\chi\tilde{t}}\tilde{a}^{-3}$. From these relations we see that during the stage $\bar{m}_\chi > \tilde{H} \gg \Gamma_\chi$ the inflaton performs damped oscillations with an energy density corresponding to a dust-like perfect fluid ($\rho_{\chi(E)} \sim \tilde{a}^{-3}$) with slow decay $\sim e^{-\Gamma_\chi\tilde{t}} \sim 1$. This reheating stage ends when $\tilde{H} \lesssim \Gamma_\chi$ and the evolution of the energy density is dominated by the exponential decrease due to decay with rate Γ_χ.

If the decay rate Γ_χ of the inflaton particles into usual Standard Matter (SM) is sufficiently large, then starting from the characteristic decay time $\tilde{t}_D \sim \Gamma_\chi^{-1}$, i.e. from the time of the most intensive reheating, the energy density of the corresponding relativistic particles behaves as $\rho_{SM(E)} \sim \tilde{a}^{-4}$. Clearly, the energy loss of the χ field due to the decay process is accompanied by a corresponding energy increase of the decay

products. As result, the effective energy density $\rho_{eff(E)} := \rho_{\chi(E)} + \rho_{SM(E)}$ is only deluted by the cosmological expansion and can be roughly approximated as

$$\rho_{eff(E)} = \rho_{\chi(E)} + \rho_{SM(E)} \sim e^{-\Gamma_\chi \tilde{t}} \tilde{a}^{-3} + (e^{-\Gamma_\chi \tilde{t}_1} - e^{-\Gamma_\chi \tilde{t}}) \tilde{a}^{-4} \sim \tilde{a}^{-3}, \tilde{a}^{-4} \,. \tag{3.33}$$

This relation holds for times $\tilde{t} \gtrsim \tilde{t}_1 \sim \bar{m}_\chi^{-1}$ with $\Gamma_\chi \tilde{t}_1 \ll 1$, where $\tilde{t}_1$ plays the role of an effective initial time which fixes the begin of the coherent χ oscillations and of the decay process. From the Einstein equations of the extended interacting system we can derive a corresponding Friedman equation similar to (3.6). Obviously, for a viable model the internal space should be stabilized before the nucleosynthesis, i.e. inequality $|\kappa_0^2 \dot{\varphi}| \ll 1$ should take place before this stage. Then, starting from this moment, the Hubble parameter is defined from this extended Friedman equation by $\rho_{eff(E)}$: $\tilde{H}^2 \approx \frac{1}{3} \kappa_0^2 \rho_{eff(E)}$. Thus, the Hubble parameter during the post-inflationary stage, including the period of nucleosynthesis, is defined by the energy densities $\rho_{\chi(E)}$ and $\rho_{SM(E)}$ and can be roughly approximated as $\tilde{H} = s/\tilde{t}$ where $s = 1/2, 2/3$ for $\rho_{eff(E)} \sim \tilde{a}^{-4}, \tilde{a}^{-3}$ respectively[4]. Hence, in the same approximation we have

$$\rho_{eff(E)} \approx \frac{3s^2}{\kappa_0^2} \tilde{t}^{-2} \,. \tag{3.34}$$

Let us now in some detail consider the required asymptotic evolution of the modulus field $\varphi \approx 0$ during the post-inflationary stage up to the period of nucleosynthesis, when the modulus stabilization should be finished. From eq. (A.13) given in the Appendix we see that during the inflaton dominated post-inflationary stage the equation of motion (3.14) for the modulus field φ can be rewritten as:

$$\ddot{\varphi} + 3\tilde{H}\dot{\varphi} = (\alpha_\chi - 2)\sigma\kappa_0 \rho_{\chi(E)} \,. \tag{3.35}$$

After this stage, the further evolutional behavior of the modulus field crucially depends on its coupling to the decay products of the inflaton field, i.e. on the coupling type to the SM fields. We will illustrate this specific model dependent feature with the help of the two most simplest examples, a model (I) where the decay products of the inflaton have a similar coupling to the modulus field like the inflaton itself, and a model (II) where the decay products are not coupled to the modulus field at all.

Model (I): Under a model with similar coupling of the inflaton field and its decay products (the SM fields) to the modulus field we understand a model where the evolution of the modulus field is defined via extension of (3.7) by an equation of motion of the type

$$\begin{aligned} \ddot{\varphi} + 3\tilde{H}\dot{\varphi} &= (\alpha_\chi - 2)\sigma\kappa_0 \rho_{\chi(E)} + (\alpha_{SM} - 2)\sigma\kappa_0 \rho_{SM(E)} \\ &\approx (\alpha_{eff} - 2)\sigma\kappa_0 \rho_{eff(E)} \,. \end{aligned} \tag{3.36}$$

Let us further assume that the required asymptotic stabilization of the modulus field is almost achieved $\varphi \approx 0$ so that eq. (3.36) can be considered as a non-homogeneous differential equation. In this approximation the general solution of (3.36) can be written as a sum of the general solution of the homogeneous eq. (3.17) and a particular solution of the non-homogeneous equation. As a solution of the homogeneous equation we can take eq. (3.19). Then, using (3.34) and (A.17) the solution of eq. (3.36) can be approximated as

$$\varphi(\tilde{t}) \approx -\frac{\dot{\varphi}_0}{3s - 1} \frac{\tilde{t}_0^{3s}}{\tilde{t}^{3s-1}} - \frac{2s\sigma}{\kappa_0} \ln \frac{\tilde{t}}{T_0} \tag{3.37}$$

with $s = 1/2, 2/3$ for $\rho_{eff} \sim a^{-4}, a^{-3}$ respectively. The time T_0 plays to role of an effective initial time which is defined from the value of the internal scale factor $b(\tilde{t})$ at the beginning of the evolutional stage described by (3.36). From eq. (3.37) we see that the used perfect fluid ansatz leads to a decompactification of the internal space. For times $\tilde{t} \gg \tilde{t}_0$, the internal scale factor behaves as $b \approx b_0 (\tilde{t}/T_0)^\gamma$, where $\gamma = 2s/(D-2)$ and e.g. $\gamma = 1/4, 1/3$ for $d_1 = 2$ and $s = 1/2, 2/3$ respectively. Eq. (3.37) shows also that for $\tilde{t} \gg t_0$ we obtain $\dot{b}/b \sim -\kappa_0 \dot{\varphi} \sim \tilde{t}^{-1}$, so that via extrapolation to our present time we would get the estimate 10^{-10}yr^{-1}. This estimate is much greater than 10^{-14}yr^{-1} following from observations. So, solution (3.37) leads to a decompactification of the internal space and does not ensure the necessary stabilization of the internal spaces at the present time. This was proved by the rule of contraries. First, we supposed that the modulus stabilization occurs and we can use the approximation (3.34). Then, we showed that our proposal is wrong because an unboundedly increasing destabilization term occurs in the solution (3.37).

Let us now consider model (II) with vanishing coupling of the decay products of the inflaton to the modulus field[5]. In this case eq. (3.35) of the modulus evolution holds also after the inflaton-dominated era.

---

[4]We note that the expansion parameter s is connected with the parameter α in the equation of state $P_{eff(E)} = (\alpha - 1)\rho_{eff(E)}$ by the relation $s = \frac{2}{d_0 \alpha}$. See also (A.15) - (A.17).

[5]But the decay products still define the dynamical behavior of the Universe.

Using the approximation (3.34) for the energy density ρ_χ of the decaying dust-like perfect fluid component we rewrite this equation with an ansatz analogous to (3.34) and $s = 2/3$ as

$$\ddot{\varphi} + 2\tilde{t}^{-1}\dot{\varphi} = -\frac{4\sigma}{3\kappa_0}\tilde{t}^{-2}e^{-\Gamma_\chi\tilde{t}}\,. \tag{3.38}$$

For simplicity of notations we introduce the abbreviation $q := 4\sigma/(3\kappa_0\Gamma_\chi)$. Then, the solution of (3.38) with internal space stabilization $\varphi(\tilde{t} \to \infty) \longrightarrow 0$ at $b \longrightarrow b_0$ and initial condition $\dot{\varphi}(\tilde{t} = \tilde{t}_0) = \dot{\varphi}_0$ can be easily found as

$$\varphi(\tilde{t}) = \left(qe^{-\Gamma_\chi\tilde{t}_0} - \dot{\varphi}_0\tilde{t}_0^2\right)\frac{1}{\tilde{t}} - q\int_{\tilde{t}}^{\infty}\frac{e^{-\Gamma_\chi t}}{t^2}\,dt\,. \tag{3.39}$$

The corresponding characteristic variation of the internal scale factor for the same initial conditions on $\tilde{t}_0$ and $\dot{\varphi}_0$ as for the estimate (3.21) reads

$$\frac{\dot{b}}{b} = -\kappa_0\sqrt{\frac{D_0-2}{d_1(D-2)}}\left[q\left(e^{-\Gamma_\chi\tilde{t}} - e^{-\Gamma_\chi\tilde{t}_0}\right) + \dot{\varphi}_0\tilde{t}_0^2\right]\frac{1}{\tilde{t}^2} \tag{3.40}$$

and gives for a decay channel of the inflaton to fermions with decay rate [31] $\Gamma_\chi \sim 10^{-12}M_{Pl}$ the estimate $\dot{b}/b\big|_{\tilde{t}\sim10^3sec,d_1=2} \sim 10^{-31}yr^{-1}$. The modulus field stabilizes at the same time at $\kappa_0\varphi|_{\tilde{t}\sim10^3sec,d_1=2} \sim 10^{-35} \ll 1$. Thus, there exists a possible dynamical stabilization scenario for a decaying inflaton field and a modulus field which is not coupled to the decay products. Clearly, this dynamical stabilization scenario is rather artificial and in general the decay products will be also functionally coupled to the modulus field what can destroy the stabilization.

Above, we considered a model with zero effective cosmological constant. However, resent observations show the existence of a positive cosmological constant $\Lambda \sim 10^{-57}\text{cm}^{-2}$ for our Universe. So, it is of interest to include such a $\Lambda-$term into our consideration. This can be easily done if we suppose that the potential $V(\chi)$ has a non-zero minimum at $\chi = 0$. Thus, for $|\chi| \lesssim M_{Pl}$ the potential reads

$$V(\chi) \approx V_0 + \frac{1}{2}m_\chi^2\chi^2\,. \tag{3.41}$$

It is clear that $\Lambda_{eff} \equiv e^{2\sigma\kappa_0\varphi}V_0$ plays the role of the effective cosmological constant which asymptotically tends to V_0 in the case of the internal space stabilization $\varphi \longrightarrow 0$. Such a behavior of the effective cosmological constant is similar to the quintessence scenario [18]. We suppose that in accordance with observations $\kappa_0^2V_0 \sim 10^{-57}\text{cm}^{-2} > 0$. For such values of V_0 the influence of V_0 on the Universe and the fields dynamics becomes essential only at a stage close to our present time. During earlier post-inflationary evolution stages $e^{2\sigma\kappa_0\varphi}V_0$ is negligible compared with $\rho_{\chi(E)}$: $e^{2\sigma\kappa_0\varphi}V_0 \ll \rho_{\chi(E)} = \frac{1}{2}\dot{\chi}^2 + \frac{1}{2}\bar{m}_\chi^2\chi^2$. But, with progression of the Universe expansion $\rho_{\chi(E)}$ decreases and becomes less and less compared with $e^{2\sigma\kappa_0\varphi}V_0$. Let us investigate the influence of V_0 on the internal space stabilization when $e^{2\sigma\kappa_0\varphi}V_0 \gtrsim \rho_{\chi(E)}$. We also assume that up to this time $|\kappa_0\varphi| \ll 1$. Under this assumptions eq. (3.35) is modified as follows:

$$\ddot{\varphi} + 3\tilde{H}\dot{\varphi} = -2\kappa_0\sigma e^{2\sigma\kappa_0\varphi}V_0 + (\alpha-2)\kappa_0\sigma\rho_{\chi(E)} \approx -2\kappa_0\sigma V_0\,. \tag{3.42}$$

A solution of this equation can be found similar to eq. (3.37) and reads

$$\varphi(\tilde{t}) = -\frac{\dot{\varphi}_0}{3s-1}\frac{\tilde{t}_0^{3s}}{\tilde{t}^{3s-1}} - \frac{\sigma\kappa_0V_0}{3s+1}\tilde{t}^2\,. \tag{3.43}$$

Thus, an effective cosmological constant results also in a decompactification (destabilization) of the internal space at late times.

Summarizing the results of the present section, we can conclude that the considered possible dynamical stabilization scenario is very sensitive to the coupling of the modulus field to small fluctuations of the inflaton and matter fields, as well as to a non-vanishing vacuum contribution. Thus, this scenario is in general not sufficiently robust and each concret model needs a detailed study whether a dynamical modulus stabilization could work or not. In the next section we extend our toy model and induce a trapping mechanism to guarantee the modulus stabilization.

4   Stable compactification

In this section we present an example for a model which ensures stable compactification of the internal space by a trapping of the geometrical modulus at the minimum of the effective potential. For this purpose, we modify our setup model of section 2 including a non-zero bare $D-$dimensional cosmological constant

Λ into the corresponding action functional (2.4), (2.7) and assume that the internal space is not Ricci-flat $R[g^{(1)}] \equiv R_1 = \text{const} \neq 0$. Then, instead of (3.4) we get an effective potential

$$
\begin{aligned}
U_{eff}[\varphi, \chi] &= e^{2\kappa_0\varphi\sqrt{\frac{d_1}{(D-2)(D_0-2)}}}\left[-\frac{1}{2\kappa_0^2}\tilde{R}_1 e^{2\kappa_0\varphi\sqrt{\frac{D_0-2}{d_1(D-2)}}} + \frac{1}{\kappa_0^2}\Lambda + V(\chi)\right] \equiv \\
&\equiv e^{2\sigma\kappa_0\varphi}\left[-\frac{1}{2\kappa_0^2}\tilde{R}_1 e^{2\gamma\kappa_0\varphi} + \frac{1}{\kappa_0^2}\Lambda + V(\chi)\right],
\end{aligned}
\tag{4.1}
$$

where $\tilde{R}_1 := R_1/b_0^2$ and we used the obvious abbreviations $\sigma = \sqrt{d_1/2(d_1+2)}$ and $\gamma = \sqrt{2/d_1(d_1+2)}$. We restrict ourselves to the case of a potential $V(\chi)$ with a single zero minimum $V(\chi_{min}) = V(\chi)|_{min} = 0$, $\partial_\chi^2 V|_{min} = m_\chi^2 > 0$ because the non-zero minimum case $V(\chi)|_{min} = V_0$ is trivially reduced to the zero minimum one by inclusion of V_0 into Λ/κ_0^2.

It is easy to check that the effective potential (4.1) has a global minimum at the point $\varphi = 0, \chi = \chi_{min}$ if the bare cosmological constant and the scalar curvature of the internal space are both negative[6]: $\Lambda < 0$, $R_1 < 0$. Additionally it is necessary that these parameters are connected with the compactification scale b_0 by a fine-tuning condition

$$
2\sigma\Lambda = (\sigma+\gamma)\frac{R_1}{b_0^2} \quad \Longleftrightarrow \quad \Lambda = \frac{D-2}{2d_1}\frac{R_1}{b_0^2}.
\tag{4.2}
$$

From the Hessian (the mass matrix) of the effective potential

$$
\begin{pmatrix} \partial_{\varphi\varphi}^2 & \partial_{\varphi\chi}^2 \\ \partial_{\chi\varphi}^2 & \partial_{\chi\chi}^2 \end{pmatrix} U_{eff}|_{\varphi=0,\chi_{min}} = \begin{pmatrix} -2\gamma(\sigma+\gamma)\tilde{R}_1 & 0 \\ 0 & m_\chi^2 \end{pmatrix}
\tag{4.3}
$$

it is clear that in contrast with (3.16) the minimum is non-degenerate and induces a non-vanishing mass $m_\varphi^2 = -2\gamma(\sigma+\gamma)\tilde{R}_1$ of the modulus field[7]. As result the position $\varphi = 0$ is energetically favored and provides the necessary trapping.

The trapping can also be seen directly from the equations of motion of the (φ, χ) system. Whereas substitution of (4.1) into the χ field equation (3.8) does not change this equation, the equation (3.7) for the modulus field reads now

$$
\ddot{\varphi} + d_0\tilde{H}\dot{\varphi} = -\frac{\partial U_{eff}}{\partial\varphi} = -e^{2\sigma\kappa_0\varphi}\left[-\frac{1}{\kappa_0}(\sigma+\gamma)\tilde{R}_1 e^{2\gamma\kappa_0\varphi} + \frac{2\sigma}{\kappa_0}\Lambda + 2\sigma V(\chi)\right].
\tag{4.4}
$$

So, for fine tuned parameters (4.2) the point $\varphi = 0$, $\chi = \chi_{min}$ is the trivial solution of the system (3.8), (4.4) and small linear perturbations around this solution satisfying

$$
\begin{aligned}
\delta\ddot{\varphi} + d_0\tilde{H}\delta\dot{\varphi} - 2\gamma(\sigma+\gamma)\tilde{R}_1\delta\varphi &= 0, \\
\delta\ddot{\chi} + d_0\tilde{H}\delta\dot{\chi} + m_\chi^2\delta\chi &= 0
\end{aligned}
\tag{4.5}
$$

are damped by the present friction terms. The global modulus dynamics is easily understood from the form of the effective potential. The minimum at $(\varphi = 0, \chi = \chi_{min})$ is the deepest point and defines after stabilization of the system the effective cosmological constant $\Lambda_{eff} \equiv \kappa_0^2 U_{eff}|_{min} = \tilde{R}_1/d_1 < 0$ of the external spacetime. In the small-scale-factor region we have $U_{eff}(\varphi \to \infty) \to \infty$ and in the decompactification region $U_{eff}(\varphi \to -\infty) \to 0$, so that we are led to the following implications. 1) the Brustein-Steinhardt [33] problem[8] cannot occur because there is no barrier in the model at all, separating the minimum of the potential from an asymptotically allowed region (the single minimum is located energetically deeper than the decompactification region); 2) higher order approximations to the solution of (4.4) will not alter the stabilization picture for the same reason; and 3) the dying away friction term $d_0\tilde{H}\dot{\varphi}$ will leave at most small fluctuations of the geometrical modulus field around the minimum. This means, that this minimum stably compactifies (traps) the internal space. Clearly, such a stabilization mechanism is different from the above discussed dynamical stabilization where the effective potential has no minimum with respect to φ.

As conclusion of this section we would like to note that the negative effective cosmological constant $\Lambda_{eff} < 0$ of the model (4.1) leads to a turning point in the scale factor evolution of the external space, i.e. a stage of external space inflation is followed be a period of contraction. This happens when the effective energy density ρ_{eff} of the fields in the Universe becomes smaller than $|\Lambda_{eff}|$. Model (4.1) with stable internal space can be brought in agreement with the observed positive effective cosmological constant of the Universe, e.g. if one takes into account additional higher dimensional form fields [34].

---

[6]Compact internal spaces with negative curvature $R_1 = -d_1(d_1 - 1)$ can be constructed, e.g., as hyperbolic coset manifolds H^{d_1}/Γ^{d_1} (for details see [32]). Here H^{d_1} is an infinite hyperbolic space and Γ^{d_1} — an appropriate group of discrete isometries. The coset manifold itself can be imagined to be built up from a fundamental polyhedron in H^{d_1} with faces pairwise identified.

[7]These massive modes (gravitational excitons [6]) propagate in the external spacetime and can yield a considerable contribution to the dark matter in our Universe [10].

[8]The Brustein-Steinhardt problem can occur for potentials with an energetically allowed decompactification region which is separated from a narrow valley around the minimum by a barrier. Supposing that the modulus field rolls down the potential it can overshoot this narrow valley and enter the decompactification region. The problem consists in the fact that the modulus field with high kinetic energy will not "feel" the valley and will not stabilize in the corresponding minimum.

5 Conclusion

In the present paper we investigated in detail the recently proposed [21] mechanism of internal space dynamical compactification in a multidimensional model with one Ricci-flat internal space and a massive scalar field acting as inflaton. As shown in section 3, such a dynamical stabilization could be possible if the modulus field is only coupled to the inflaton but not to its decay products. Due to the exponential decay of the inflaton during reheating the force term in the modulus equation, obtained from the effective potential, decreases also exponentially so that the modulus field can dynamically stabilize via friction. If the decay products of the inflaton will couple similar to the modulus field like the inflaton itself, the dynamical stabilization will not occur. An analogous decompactification has place for a model with non-vanishing effective cosmological constant. A stable compactification for the considered model can be ensured via trapping of the internal space scale factor by a minimum of the effective potential. An example of such type of stable compactification is given in section 4. Oscillations of the modulus and the inflaton field around the minimum are observed as massive scalar particles (gravexitons and inflaton particles) in the external spacetime. Such particles were considered in the electroweak and the planckian fundamental scale approaches in Ref. [10]. In the same paper it was pointed out that there exists a rescaling for gravexciton masses in the different approaches and extremely light particles may arise in the planckian fundamental scale approach. Further it was shown there that particles with masses $m_\varphi \sim 10^{-33}$eV are of special interest because, first, they do not overclose the Universe, second, the period of their oscillations $T_{osc} \sim 1/m_\varphi$ is of order of the Universe age $\sim 10^{18}$sec, and third, the effective cosmological term $\Lambda_\varphi \equiv \kappa_0^2 m_\varphi^2 \varphi^2 \sim 10^{-57} cm^{-2}$ corresponding to models with such particles and $\varphi \sim M_{Pl}$, takes a value of order of the presently observable cosmological constant in the Universe. Thus, these particles evolve extremely slowly within their minimum and it is attractive to treat Λ_φ as an effective cosmological constant in the spirit of quintessence (for models with zero minimum of an effective potential). However, such a quintessence would have a drawback because it requires a highly fine tuned initial condition of the modulus field φ (see e.g. [35]).

A   The effective potential of a higher dimensional perfect fluid

In this Appendix we assume that the bulk scalar field $\tilde{\chi}$ behaves like a perfect fluid living in the $D-$dimensional bulk spacetime and obeying the equation of state

$$P_{\tilde{\chi}} = (\alpha - 1)\rho_{\tilde{\chi}} \tag{A.1}$$

with $0 \leq \alpha \leq 2$ a constant. The energy density $\rho_{\tilde{\chi}} \equiv -T_0^0$ of the scalar field $\tilde{\chi}$ and its pressure $P_{\tilde{\chi}} \equiv T_N^N$ are defined from the action functional (2.4). Further, we assume that the metric of the D_0-dimensional external spacetime is given by the ansatz (2.20) and that a homogeneous approximation for the scalar fields $\tilde{\chi} = \tilde{\chi}(t), \Phi = \Phi(t)$ can be used. Then, as it was shown in [25], the action (2.4) of the gravity — scalar field system is equivalent to the gravity — perfect fluid action

$$S = \frac{1}{2\kappa_D^2} \int_M d^D x \sqrt{|g|} \left\{ R[g] - 2\frac{\kappa_D^2}{v_{d_1}} \rho_\chi(a,b) \right\} , \tag{A.2}$$

where

$$\rho_\chi(a,b) = \frac{A}{a^{\alpha d_0} b^{\alpha d_1}} , \qquad A := const \tag{A.3}$$

and we took into account the redefinitions (2.9) of the scalar field $\tilde{\chi} \longrightarrow \chi$ and the potential $U(\tilde{\chi}) \longrightarrow V(\chi)$. Dimensional reduction of (A.2) gives the Brans-Dicke frame action functional similar to (2.11)

$$S = \int_{M_0} d^{D_0} x \sqrt{|g^{(0)}|} \left\{ \Phi R \left[g^{(0)} \right] - \frac{\omega}{\Phi} g^{(0)\mu\nu} \partial_\mu \Phi \, \partial_\nu \Phi - 2\kappa_0^2 \Phi \rho_{\chi(BD)}(a,\Phi) \right\} , \tag{A.4}$$

where the energy density $\rho_{\chi(BD)}$ is defined as

$$\rho_{\chi(BD)}(a,\Phi) = \frac{1}{2}\left(\frac{d\chi}{dt}\right)^2 + V(\chi) \tag{A.5}$$

$$= \frac{A}{a^{\alpha d_0}\,(2\kappa_0^2\Phi)^\alpha\,b_0^{\alpha d_1}}\,. \tag{A.6}$$

Finally, via conformal transformation (3.1) we pass to the Einstein frame and arrive at

$$S = \frac{1}{2}\int_{M_0} d^{D_0}x\sqrt{|\tilde{g}^{(0)}|}\left\{\frac{1}{\kappa_0^2}\tilde{R}\left[\tilde{g}^{(0)}\right] - \tilde{g}^{(0)\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - 2\rho_{\chi(E)}(\tilde{a},\varphi)\right\}\,. \tag{A.7}$$

The energy density $\rho_{\chi(E)}(\tilde{a},\varphi)$ can be recast as

$$\rho_{\chi(E)}(\tilde{a},\varphi) = \frac{1}{2}\left(\frac{d\chi}{d\tilde{t}}\right)^2 + e^{2\sigma\kappa_0\varphi}V(\chi) \tag{A.8}$$

$$= e^{2\sigma\kappa_0\varphi}\rho_{\chi(BD)}(a,\Phi) \tag{A.9}$$

$$= e^{(2-\alpha)\sigma\kappa_0\varphi}\tilde{\rho}_\chi(\tilde{a})\,, \tag{A.10}$$

where we used the relations between the synchronous times $d\tilde{t} = \pm e^{-\sigma\kappa_0\varphi}dt$ and the scale factors $\tilde{a} = e^{-\sigma\kappa_0\varphi}a$ in the Brans-Dicke frame (t,a) and the Einstein frame $(\tilde{t},\tilde{a})$ and defined the reduced energy density $\tilde{\rho}_\chi$ as

$$\tilde{\rho}_\chi(\tilde{a}) \equiv \frac{A}{\tilde{a}^{\alpha d_0}b_0^{\alpha d_1}}\,. \tag{A.11}$$

The equations of motion following from action functional (A.7) for homogeneous field $\varphi = \varphi(\tilde{t})$ and energy density $\rho_{\chi(E)} = \rho_{\chi(E)}(\tilde{t})$, and Ricci-flat external space $R[\bar{g}^{(0)}] = 0$ read:

$$\tilde{H}^2 \equiv \left(\frac{\dot{\tilde{a}}}{\tilde{a}}\right)^2 = \frac{2\kappa_0^2}{d_0(d_0-1)}\left\{\frac{1}{2}\dot{\varphi}^2 + \rho_{\chi(E)}(\tilde{a},\varphi)\right\}\,, \tag{A.12}$$

$$\ddot{\varphi} + d_0\tilde{H}\dot{\varphi} = (\alpha-2)\sigma\kappa_0\rho_{\chi(E)}(\tilde{a},\varphi)\,, \tag{A.13}$$

$$\dot{\tilde{H}} = -\frac{\kappa_0^2}{d_0-1}\left\{\dot{\varphi}^2 + \alpha\rho_{\chi(E)}(\tilde{a},\varphi)\right\}\,. \tag{A.14}$$

Eqs. (A.10) and (A.13) show that for $\alpha = 2$ the interaction between the modulus field φ and the perfect fluid (scalar field χ) is absent. This fact is easily explained because $\alpha = 2$ corresponds to a perfect fluid with ultra-stiff equation of state, which describes a scalar field with vanishing potential energy $V(\chi) \equiv 0$. The other extremal case with $\alpha = 0$ describes a vacuum equation of state and can be used for potentials $V(\chi)$ with non-zero minimum V_0.

At the end of this Appendix let us consider a model with stabilizing modulus field $\varphi \approx 0$ and decaying field χ. Similar to model (II) of section 3 we assume that the decay products with energy density $\rho_{SM(E)} \sim \tilde{a}^{-\alpha d_0}$ are not coupled to the modulus field, but that they, nevertheless, define the dynamics of the external space. Then the corresponding equation system follows from (A.12), (A.14) and reads

$$\tilde{H}^2 = \frac{2\kappa_0^2}{d_0(d_0-1)}\rho_{SM(E)}(\tilde{a})\,, \tag{A.15}$$

$$\dot{\tilde{H}} = -\frac{\alpha\kappa_0^2}{d_0-1}\rho_{SM(E)}(\tilde{a})\,, \tag{A.16}$$

so that the Hubble parameter behaves as $\tilde{H} = s/\tilde{t}$ with

$$s = \frac{2}{d_0\alpha}\,. \tag{A.17}$$

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